Embodied construction grammar as layered modal languages

Anders Søgaard
Center for Language Technology
Njalsgade 80
DK-2300 Copenhagen
`anders@cst.dk`

May 31, 2006

0.1 Layering

- Say a theory of D and a theory of E, two mutually exclusive domains, are one-way dependent parts of a theory of the union of the two domains. It holds that
    - $\delta \vee \epsilon$
    - $\neg(\delta \wedge \epsilon)$
    - $\langle\tau\rangle\phi \rightarrow \delta \wedge \langle\tau\rangle(\epsilon \wedge \phi)$
- If the parts are interdependent, introduce/define a converse modality.
- This is a simple model of layering in which two logical theories are layered. Since our example is very simple, the two logics form a standard modal logic, but the method is the same: The axiom sets of the two logics are translated into domain-specific axioms of the union logic, and their internal relations defined (as in the above).

0.2 Grammars as layered theories

- Grammars are often modular, i.e. they identify dependent domains, and thus qualify for layered descriptions; in particular, if these domains are treated in formally different ways. Obvious examples in the literature include GPSG and LFG, which both identify constituent domains and functional domains.

- Layering is necessary for scalability for two reasons: Layering is an expansion method, and layering enables local efficiency in the context of automated reasoning.

0.3 Embodied construction grammar

- ECGs are written up in a representational language of attribute-value structures. An attribute-value structure of a signature of attribute labels and value labels (sometimes called atoms) is a 3-tuple of a set of nodes, a set of (labelled) partial functions and a set of (labelled) unary ones.
- ECG comprises several domains: schemas, constructions, and mental spaces.
- Parsing is either unification-based or model-theoretic.

$$\begin{bmatrix} \textsc{funct} & \begin{bmatrix} \textsc{num} & \boxed{1} \end{bmatrix} \\ \textsc{arg} & \begin{bmatrix} \textsc{num} & \boxed{1} \end{bmatrix} \end{bmatrix}$$

$$\models \langle\textsc{funct}\rangle\langle\textsc{num}\rangle i \wedge \langle\textsc{arg}\rangle\langle\textsc{num}\rangle i$$

schema Trajector-Landmark
subcase of Image-Schema
roles
trajector: *a*
landmark: *b*

is equivalent to

$$\begin{bmatrix} \textit{trajector\_landmark} & \\ \textsc{roles} & \begin{bmatrix} \textsc{trajector} & a \\ \textsc{landmark} & b \end{bmatrix} \end{bmatrix}$$

$$\wedge \textit{trajector\_landmark} \sqsupseteq \textit{image\_schema}$$

- $\mathit{schema} \vee \mathit{constr} \vee \mathit{ms}$
- $\neg(\mathit{schema} \wedge \mathit{constr} \vee \mathit{schema} \wedge \mathit{ms} \vee \mathit{constr} \wedge \mathit{ms}$
- $\langle \mathit{map} \rangle \phi \rightarrow (\mathit{schema} \wedge \langle \mathit{map} \rangle (\mathit{schema} \wedge \phi)) \vee (\mathit{ms} \wedge \langle \mathit{map} \rangle (\mathit{ms} \wedge \phi))$

Reentrancies are denoted by $\leftrightarrow$. In other words,

$$\boxed{\text{a} \leftrightarrow \text{b}}$$

corresponds to

$$\begin{bmatrix} \text{A} & \boxed{1} \\ \text{B} & \boxed{1} \end{bmatrix}$$

Feature structures, more generally, are governed by a number of axioms; see for instane [Wed97]. Feature structures have (mostly) deterministic attributes, and they are acyclic, connected and rooted.

(1) $\langle \alpha \rangle i \rightarrow [\alpha] i$ (functionality)

(2) $i \rightarrow \neg \langle \alpha_1 \cup \cdots \cup \alpha_n \rangle^+ i$ (acyclicity)

(3) $E(\mathit{root} \wedge \neg \langle \alpha_1^{-1} \cup \cdots \cup \alpha_n^{-1} \rangle \top) \wedge A(\langle \alpha_1^{-1} \cup \cdots \cup \alpha_n^{-1} \rangle^* \mathit{root})$ (connectedness and rootedness)

(R_α is any relation in $\mathcal{R}$, $\mathcal{R} = \{\alpha_1 \ldots \alpha_n\}$, and axioms are multiplied in the set of nominals.)

The **evokes** operator triggers a new schema and allows reference to it. This corresponds to existential quantification and naming, e.g.

$$\boxed{\begin{array}{l}\textbf{schema}\ \mathrm{s}\\ \quad\textbf{evokes}\ \mathrm{t}\ \textbf{as}\ \mathrm{i}\\ \qquad\textbf{roles}\\ \qquad\quad \mathrm{k} \leftrightarrow \mathrm{i.l}\end{array}}\ \boxed{\begin{array}{l}\textbf{schema}\ \mathrm{t}\\ \quad\textbf{roles}\\ \qquad \mathrm{l} :\ a\end{array}} \models s \wedge \langle k\rangle j \wedge E(i \wedge \langle l\rangle(a \wedge j))$$

The other complication is the "::" constraints.

It is important to note that **evokes** and constituent structure are **the only recursive parts of the feature geometry**.

0.4 Parsing

[Bry04] is unification-based, but our algorithm is entirely model-theoretic. Our LKB implementation mimics [Bry04]. Model-theoretic algorithms have a number of advantages (declarativity, interlingual comparison, flexibility, linearization, property inheritance), but also *scalability.* Our algorithm:

1. The string is encoded in our logic and conjoined with the axioms of ECG.
2. If this conjoined formula is satisfiable, the string is accepted.

Existential quantification and an equivalent of the Immediate Dominance Principle ensure parsing.

0.5 Inference

Only three domains are included here, for simplicity. The domains include a referential (R), a spatial (S), and a temporal one (T). The transitions are called R_{r2s}, from the referential to the spatial domain, and R_{s2t}, from the spatial to the temporal domain. The domains are denoted by propositions *ref*, *space* and *time*.

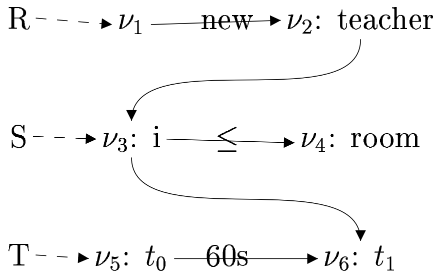
Example 0.1. Our layering is illustrated by this simple example:

(4) The new teacher enters the room in a minute.

Think of the room as a spatial region that the teacher at some point (in time) is part of. t_0 is the time of the utterance.

$$\begin{aligned}
& E(\mathit{time} \wedge t_1 \wedge @_{t_0}\langle \mathit{60s}\rangle t_1) \\
\wedge \quad & E(\mathit{space} \wedge i \wedge \leq \mathit{room} \wedge @_i\langle \mathit{s2t}\rangle t_1) \\
\wedge \quad & E(\mathit{ref} \wedge \langle \mathit{new}\rangle(\mathit{teacher} \wedge \langle \mathit{r2s}\rangle i))
\end{aligned}$$

The model that satisfies the formula will look like this:



The curved lines are the projections from one domain to another, i.e. R_{r2s} and R_{s2t}. The R domain says that there is a teacher who is new. The S domain says that his spatial instantiation is part of the spatial region that is occupied by the room at time t_1. The T domain tells us that t_1 is 60 seconds from now.

Remark 0.2. A remark on the :: constraints. In [CFPS02, Figure 2], two such constraints are used to say that in the schema "Translational-Motion", the mover's location changes from that of the source to that of the goal at the time of the motion, i.e. because of the motion. Call this temporal state t_m. The constraints

...
before :: mover.location $\leftrightarrow$ source
after :: mover.location $\leftrightarrow$ goal

thus translate into

$$A(\langle < \rangle t_m \rightarrow \langle s2t^{-1} \rangle source) \text{ and}$$
$$A(\langle > \rangle t_m \rightarrow \langle s2t^{-1} \rangle goal)$$

0.6 Computational issues

Since only the constituent structure and the **evokes** operator are recursive, and since there are no unary projections in ECG, it is easy to see that in the absence of **evokes**, the maximum size of the (smallest) model of some σ is

$$(2|\sigma| - 1) \times \textbf{paths} \text{ where } \textbf{paths} = |\{\pi \in \texttt{Lbls}^* | \text{no label occurs twice in } \pi\}|$$

Since any modal/hybrid/dynamic formula ϕ (in the absence of quantification over nominals) that represents an **evokes**-free ECG recognition problem for a string of c length, can be evaluated in a model of size polynomial to c, a suitable model can be non-deterministically chosen, and ϕ is evaluated in polynomial time, since model checking in such languages is decidable in polynomial time [FdR06]. It follows that

Theorem 0.3. *The universal recognition problem of* ***evokes****-free ECG is decidable in non-deterministic polynomial time.*

Remark 0.4. The result in Theorem 0.3 transfers to unification categorial grammar and construction grammar where unary rules are bound somehow (offline-parsability).

Remark 0.5. A full NP-completeness result is relatively easy to establish by reduction of 3SAT.

Corollary 0.6. *If the* ***evokes*** *operator is polynomially bound, ECG is still in NPTIME.*

0.7 Implementation

schema s
evokes t **as** i
roles
k ↔ i.l

schema t
roles
l : *a*

is equivalent to

$$\begin{array}{c} \begin{bmatrix} s & \\ \text{ROLES } \boxed{1} & \\ \text{EVOKES} & \begin{bmatrix} t & \\ \text{ROLES} & \begin{bmatrix} \text{L } \boxed{1} \end{bmatrix} \end{bmatrix} \end{bmatrix} \\ \Big| \\ \begin{bmatrix} s \\ \text{ROLES } \boxed{1} \text{ a} \end{bmatrix} \end{array}$$

The **evokes** operator is thus replaced by a recursive attribute which embeds the new structure. Since the EVOKES attribute is defined for this sole purpose, it is really no different from the disconnected substructure intended in the ECG-style notation. The other complication, the :: operator, deserves a brief remark too. If the relational constraints of the schemas are augmented with events, that is, each proposition is defined relative to some event (as in Davidsonian semantics), the ::-style constraints can be translated into ordinary relations. This extends the feature geometry a bit, of course, but enables us to implement ECG in LKB.

References

[Bry04] John Bryant. Scalable construction-based parsing and semantic analysis. Presented at Scalable Natural Language Understanding 2004, 2004.

[CFPS02] Nancy Chang, Jerome Feldman, Robert Porzel, and Keith Sanders. Scaling cognitive linguistiscs: formalisms for language understanding. Presented at Scalable Natural Language Understanding 2002, 2002.

[FdR06] Massimo Franceschet and Maarten de Rijke. Model checking for hybrid logics. *Journal of Applied Logic*, 2006. In press.

[Wed97] Jürgen Wedekind. Approaches to unification in grammar. In Patrick Blackburn and Maarten de Rijke, editors, *Specifying syntactic structure*, pages 245–280. CSLI Publications, Stanford, California, 1997.